TAUP-2688/07
hep-th/yymmnnn

Non critical holographic models of the thermal phases of QCD

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Abstract

We analyze the thermal phases of a non critical holographic model of QCD. The model is based on a six dimensional background of N_c non extremal D4 branes wrapping a spacial circle of radius R and the compactified Euclidean time direction of radius $\beta = 1/T$. We place in this background stacks of N_f D4 and anti-D4 flavor probe branes with a separation distance L at large radial direction. The analysis of the DBI effective action yields the following phase diagram: At low temperature the system is in a confining phase with broken chiral symmetry. In the high temperature deconfining phase chiral symmetry can be either restored for $L > L_c = 1.06R$ or broken for $L < L_c$. All of these phase transitions are of first order. We analyze the spectrum of the low-spin and high-spin mesons. High spin mesons above certain critical angular momentum "melt". We detect (no) drag for (mesons) quarks moving in hot quark-gluon fluid. The results resemble the structure and properties of the thermal Sakai-Sugimoto model derived in hep-th/0604161.

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1 Introduction

Recently the phases of thermal holographic QCD (HQCD) have been analyzed [1] in the context of the model of Sakai and Sugimoto [2, 3]. It was found out that the confinement/deconfinement and chiral symmetry breaking/restoring phase transitions are first order transitions and they do not necessarily coincide with each other. The system may admit an intemediate deconfined phase with broken chiral symmetry.

The mesonic world at these phases was later investigated in [4]. The temperature dependence of low-spin as well as high-spin meson masses was shown to exhibit a pattern familiar from the lattice. The Goldstone bosons associated with chiral symmetry breaking were shown to disappear above the chiral symmetry restoration temperature. The dissociation temperature of mesons as a function of their spin was determined, showing that at a fixed quark mass, mesons with larger spins dissociate at lower temperatures. It was further shown that unlike quarks, large-spin mesons do not experience drag effects when moving through the quark gluon fluid. They do, however, have a maximum velocity for fixed spin, beyond which they dissociate.

HQCD models based on critical string theories suffer from the major drawback of incorporating undesired KK modes. Whereas the modes associated with the S^5 in the string theory on $AdS_5 \times S^5$ are essential to describe the dual $\mathcal{N} = 4$ SYM theory, the KK modes in models of HQCD do not correspond to modes of the gauge theory. Moreover, the mass scale of those KK modes is the same as that of the glueballs and hadrons and there is no known method to disentangle the two scales. The most natural way to overcome this problem is to consider strings in non-critical dimensions so as to minimize the set of KK modes. Since the pioneering paper of Polyakov [5]. there have been many attempts to write down an non-critical string model of QCD [6]- [10]. The main problem with non-critical holography is the fact that the corresponding SUGRA backgrounds have curvature of order one and there is no way to go to a region of small curvature by taking the limit of large λ_{tHooft}. However, it turns out that in fact any HQCD model even those based on critical string theories must have eventually curvature of order one. This is necessary to avoid a gap in the masses of low spin mesons holographically described by fluctuations of the flavor D-branes, and high spin mesons [23]. There is yet another reason in favor of backgrounds with curvature which is not small. To recast a non-trivial $a-c$ anomaly which characterizes supersymmetric QCD models, one must turn on higher curvature terms. [12], [13].

In this paper, we look at the non-critical AdS_6 black hole solution [6]. This model

was shown [7] to reproduce some properties of the 4-dimensional non-supersymmetric YM theory like an area law for the Wilson loop, a mass gap in the glueball spectrum etc. At high energies the theory is dual to a thermal gauge theory at the same temperature as the black hole temperature and at low energies the dual theory is effectively 4-dimensional pure YM. For any non-critical model, the curvature is of order one in units of α' and hence higher order curvature correction may affect the structure of the model. On the other hand unlike in [2] where in what corresponds to the "uv region" the dilaton blows up and one has to elevate the model to an M theory setup, the non-critical model admits small small string coupling and hence stringy corrections can be safely ignored.

In this work we introduce flavor in this setup by adding D4,$\overline{\text{D4}}$ probe branes. This is similar to the D8,$\overline{\text{D8}}$ of the Sakai Sugimoto model in critical dimension. We analyze the classical configurations of the probe branes. At the low temperature phase the only solution of the equations of motion is a U shape, where the branes and anti-branes merge together in the region that corresponds in the dual gauge theory to the IR. The U shape is a geometrical manifestation of chiral symmetry breaking. At the high temperature (deconfining) phase there are two possible solutions , again the U shape configuration and a | | shape of parallel branes and anti-branes. To determine the phase structure we compute the difference of the free energy between these two configurations. The free energy is proportional to the value of effective action. The latter includes the DBI action and a Cern Simons term of the form $\int c_5$ for charged probe branes and only the DBI term for charge-less ones. It turns out that in the former case the difference of the free energies diverges and hence it cannot correspond to the difference of the free energy between two phases of the dual gauge theory. We therefore set this CS term to zero and perform from thereon all the computations with out this CS term. This resembles the situation in other non-critical models with flavor branes like [8] and [11].

The outcome of the model is similar to that of the thermal Sakai-Sugimoto model [1]. There are three different phases. In the *low-temperature* phase the background is the Euclidean continuation of a Lorentzian background. Gluons are confined in this phase. After the confinement/deconfinement transition for the gluons, there is the *intermediate-temperature* phase. In this phase gluons are deconfined, but chiral symmetry is still broken. Mesonic bound states still exist, as the D4-brane embedding is not yet touching the horizon. At sufficiently high temperature, the lowest-energy configuration of the D4-branes is the one in which they are parallel and fall down to the horizon. This is the *high-temperature* phase, in which chiral symmetry is restored. If the ratio $L/R > 1.06$, there is no intermediate-temperature phase, so that the confinement/deconfinement and

the chiral symmetry breaking transition coincide.

We also analyze the spectrum of low-spin and high-spin mesons. Low-spin mesons correspond on the string theory side to fluctuations of the massless fields on the probe branes. We identify the Goldstone boson associated with the chiral symmetry breaking. High-spin mesons, as for the critical case, can be described as classical spinning open strings. The temperature dependence for both low-spin and high-spin mesons is similar, that is the masses of mesons go down as the temperature goes up. For high-spin mesons there is a maximum value of angular momentum beyond which mesons cannot exist and have to melt. We find the drag force that a quark experiences moving through a hot gluon plasma, and we also find that high-spin mesons do not experience any drag force because for high-spin mesons at finite temperature, one can find generalized solutions where the meson moves with linear velocity, rigidly, with free boundary conditions in the direction of motion. Hence one does not need to apply any force to maintain this motion.

A main goal of this work has been to compare the phase diagram that follows from a critical holographic model versus that one associated with a non-critical HQCD model. Since the latter is characterized by a curvature of order one, strictly speaking one is not allowed to ignore the higher curvature corrections of the supergravity action. Hence there is priori no reason that the structure of the thermal phases that emerge from our analysis will resemble at all the one extracted from a critical HQCD model. However, the outcome of this paper is that in fact the results from the critical and non-critical holographic models are very similar.

We begin in section 2 with a short review of the SS model and its behavior at finite temperature. In section 3 we describe the AdS_6 model at zero temperature. In section 4 we discuss the behavior of this theory at finite temperature. We discuss the bulk thermodynamics, which leads to the confinement and deconfinement phases, and chiral symmetry restoration at a certain temperature. In section 5 we have a close look on the spectrum of low-spin as well as high-spin mesons in different phases and also discuss the drag force on quarks and mesons.

2 Review of Sakai-Sugimoto model at finite temperature

The Sakai-Sugimoto model [2, 3] is based on a D4/D8-$\overline{\text{D8}}$ brane system consisting of N_c D4-branes compactified on S^1 and N_f D8-$\overline{\text{D8}}$-brane pairs transverse to the S^1. The brane

configuration of the system is

	t	x_1	x_2	x_3	x_4	x_5	θ_1	θ_2	θ_3	θ_4
D4	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$					
D8-$\overline{\text{D8}}$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$

with x_4 and θ's being coordinates of S^1 and S^4 respectively.

We look at the D4-branes in the large N_c and near horizon limits. In these limits they are classical solutions of the type IIA supergravity in ten dimensions. This gravitational background is dual to a five-dimensional gauge theory, which looks four-dimensional at energy scale below the compactification scale. Imposing periodic boundary conditions on the bosons and antiperiodic ones on the fermions along the compactified direction, supersymmetry is explicitly broken. The scalars and the fermions on the D4-branes become massive and are decoupled from the system at low energy. Thus one obtains a $U(N_c)$ pure gauge theory. To describe quarks in the fundamental representation of the gauge group $U(N_c)$ one introduces flavor N_f $D8-\overline{D8}$ pairs into the D4 background. We assume $N_f << N_c$ which allows us to treat the N_f $D8-\overline{D8}$ branes as probes.

The finite temperature behavior of the Sakai-Sugimoto model was discussed in [1,4,14, 15]. As opposed to the zero temperature case there are two solutions at finite temperature, because one Wick-rotates the metric (generates a black hole solution) and an asymptotic symmetry between compactified Euclidean time coordinate (with periodicity $\beta = 1/T$) and x_4 (with periodicity $2\pi R$) appears. In [1] it was shown that one of them dominates at low temperatures and the other one at high temperatures. A phase transition between these backgrounds occurs at the temperature $T_c = 1/2\pi R$. This phase transition is of the first order and represents a confinement/deconfinement transition [16].

The bulk background geometry at low temperature is represented by the following metric

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{\frac{3}{2}} \left(dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2\right) + \left(\frac{R_{D4}}{u}\right)^{\frac{3}{2}} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$e^{\phi} = g_s \left(\frac{u}{R_{D4}}\right)^{\frac{3}{4}}, \qquad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad f(u) = 1 - \frac{u_\Lambda^3}{u^3}, \tag{2.1}$$

where $d\Omega_4^2$ is the metric of S^4 and $R_{D4}^3 = \pi g_s N_c l_s^3$ with g_s and l_s being the string coupling and the string length. ϵ_4 and V_4 are the volume form and the volume of S^4. The x_4-u submanifold has a cigar-like form with a tip at $u = u_T$. To avoid singularity at the tip

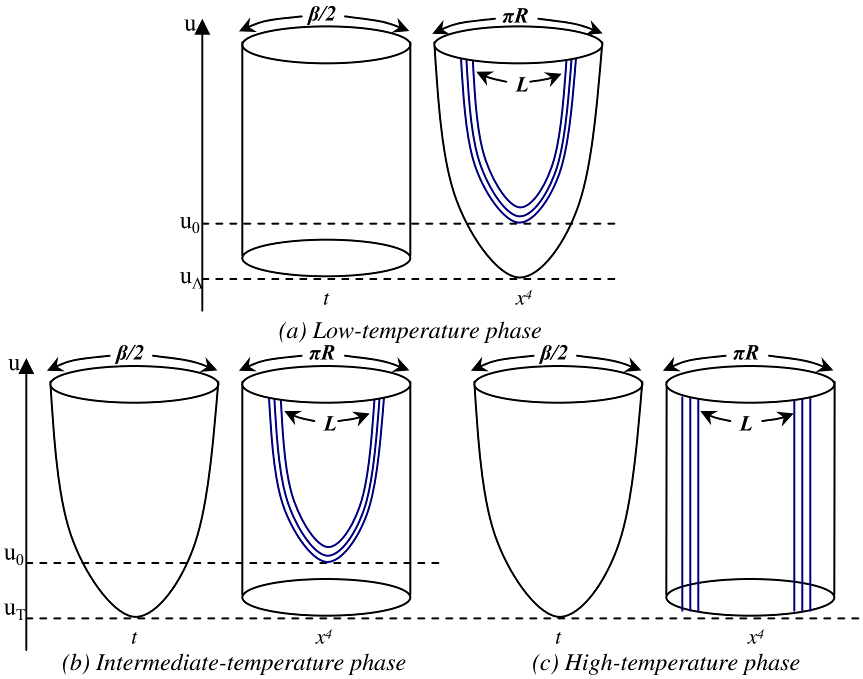

Figure 1: The configuration of flavor branes at three phases: (a) low-temperature phase, (b) intermediate and (c) high-temperature phases.

of the cigar x_4 should be periodic with periodicity

$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_\Lambda}\right)^{1/2} = 2\pi R \tag{2.2}$$

The effective action of the D8-branes consists of the DBI action and the Chern-Simons term

$$S_{\rm D8} = T_8 \int d^9x\, e^{-\phi}\,{\rm Tr}\sqrt{\det(g_{MN} + 2\pi\alpha' F_{MN})} - \frac{i}{48\pi^3}\int_{\rm D8} C_3\,{\rm Tr}F^3, \tag{2.3}$$

where g_{MN} and F_{MN} are the induced metric and the field strength on the D8-brane and T_8 is the tension of the D8-brane. The CS term in the D8-action does not affect the solution of the equation of motion of the gauge field since it has a classical solution of a vanishing gauge field.

The Hamiltonian of the action does not depend on x_4 and therefore the equation of motion equals to a constant. To solve it we assume that there is a point u_0 where the profile $u(x_4)$ has a minimum ($u'|_{u=u_0} = 0$). The form of the profile is drawn in figure 1(a).

The x_4 circle shrinks to zero at $u = u_\Lambda$. Therefore D8 branes and antibranes have no place to end and should stay all the time connected. Because of this configuration the chiral symmetry $U(N_f)_L \times U(N_f)_R$ on the probe D8-D8 pairs is always broken to a diagonal subgroup $U(N_f)_V$ in the low temperature phase.

In the high temperature phase the preferred background is the one with the interchanged role of the t and x_4 circles (by moving the factor of $f(u)$ in (2.1) from the dx_4^2 term to the dt^2 term). Now the t-circle shrinks to zero at u_T (which is now related to T rather that to R), while the x_4 circle never shrinks. While the configuration with connected flavor branes is still possible, a new configuration with parallel branes appears (see figure 1(b) and 1(c)). This new configuration is also a solution of the equation of motion of the new background DBI action and it means that chiral symmetry $U(N_f)_L \times U(N_f)_R$ is restored.

Both configuration are possible at the high temperature phase (deconfinement phase). To see when they are preferred we need to compute their free energy (the one with the lower free energy is preferred in the given temperature range). In [1] it was found that in the range $T_c \leq T < T_{\chi SB}$ ($T_{\chi SB} = 0.154/L$, L is the separation distance between the flavor branes at $u \to \infty$) the preferred configuration is with connected branes and at $T \geq T_{\chi SB}$ - with parallel branes. Therefore deconfinement and chiral symmetry restoration do not occur together but there is an intermediate phase with deconfinement and broken chiral symmetry.

3 Near extremal AdS_6 model with flavor branes at zero temperature

We are interested in the non-critical flavored version of the model of [2], which was considered in [6], [9]. The starting point is to consider unflavored conformal AdS_6 background, which is the dual of a fixed point non-supersymmetric 5-dimensional gauge theory without fundamental quarks. The construction of the model can be made by either first taking the near extremal limit of the AdS_6 background and then adding flavors, or by adding flavors first and then taking the near extremal limit of the flavored AdS_6. We follow the former one. The non-critical version of the near horizon limit of N_c near extremal $D4$-branes wrapped over a circle with anti-periodic boundary conditions takes the form of a static black hole embedded inside AdS_6. The only surviving fermionic degrees of freedom are excited Kaluza-Klein modes because the anti-periodic boundary conditions project

massless fermions out of the spectrum. At high energies the system is dual to a thermal gauge theory at the same temperature as the black hole temperature and at low energies the KK modes can not be excited and the dual theory is effectively 4-dimensional pure YM.

The 6-dimensional background metric, 6-form field strength and constant dilaton are given by

$$
\begin{aligned}
ds_6^2 &= \left(\frac{u}{R_{AdS}}\right)^2 (-dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)} \\
F_{(6)} &= Q_c \left(\frac{u}{R_{AdS}}\right)^4 dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge dx_4 \\
e^\phi &= \frac{2\sqrt{2}}{\sqrt{3}Q_c} \qquad R_{AdS}^2 = \frac{15}{2} \qquad f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^5
\end{aligned}
\tag{3.4}
$$

The space spanned by u and x_4 has a topology of a cigar with the minimum value u_Λ at its tip. To avoid a conical singularity at the origin, x_4 needs to be periodic with periodicity

$$
x_4 \sim x_4 + \frac{4\pi R_{AdS}^2}{5u_\Lambda} = x_4 + 2\pi R \tag{3.5}
$$

The typical mass scale below which the theory is effectively 4-dimensional is

$$
M_\Lambda = \frac{2\pi}{\delta x_4} = \frac{5}{2}\frac{u_\Lambda}{R_{AdS}^2} \tag{3.6}
$$

Since the gauge theory is not supersymmetric there are two ways to add flavor to the AdS_6 black hole background - by adding D4- or D5-probe branes. But it seems natural to include probe $D4$-branes and antibranes extended along the Minkowski directions and stretching to infinity in the radial direction since then the low energy limit of the gauge theory will contain massless fundamental quarks, while adding D5-branes, which need to wrap the S^1, due to the antiperiodic boundary conditions on S^1 will generate mass to the quarks of the 4-dimensional gauge theory. When all the quarks are massless one can reproduce a spontaneous chiral symmetry breaking in terms of the string dual theory. The brane configuration looks the following way

	t	x_1	x_2	x_3	x_4	x_5
D4	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	
D4-$\overline{\text{D4}}$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		$\diamond$

In the limit of large N_c and very small g_s (with fixed $g_s N_c$), $N_f \ll N_c$ and $L \gg l_s$, the coupling of the strings stretching between two D4-probe branes, two D4-probe antibranes or between a D4-probe brane and an antibrane goes to zero and they become non-dynamical sources. Hence, the degrees of freedom in the low energy limit and in the above limiting case are described by the strings stretching between color branes or between a color brane and a probe brane/antibrane. The gauge symmetry of the flavor branes $U(N_f) \times U(N_f)$ becomes a global symmetry of QCD and represents a chiral symmetry of the quarks. The fermions that appear from the color - D4-probe branes intersection transform as $(\bar{N}_f, 1)$ of the global symmetry and that from the color - $\overline{D}4$-probe branes intersection transform as $(1, \bar{N}_f)$. Both fermions transform in the fundamental N_c representation of the color group.

The picture is similar to the Sakai-Sugimoto model [1, 3] but the background we consider is non-critical. For any non-critical model, the curvature is of order one in units of α'. But taking large N_c limit guarantees small string coupling and one expects that stringy corrections will not affect calculations on the non-critical gravity side. The results in [7] are at least of the same order of magnitude as those given by experiments or lattice calculations, showing therefore that this assumption is not meaningless.

We consider the action

$$S_{D4} = T_4 \int d^5 x e^{-\phi} \sqrt{-det\hat{g}} - \tilde{a} T_4 \int \mathcal{P}(C_{(5)}) \tag{3.7}$$

where $\hat{g}$ is the induced metric over D4-brane worldvolume and $\mathcal{P}(C_{(5)})$ is the pull-back of the RR 5-form potential over the D4-brane worldvolume. Taking $\tilde{a}$ is a constant which fixes the relative strength of the DBI and CS terms in (3.7). $\tilde{a}$ should be taken equal to zero if WZ coupling is not present at all. $\tilde{a}$ equals to one would be the direct naive generalization from the 10-dimensional theory, but it is not well understood what should be written in this two-derivative approximation to the non-critical setup.

The induced metric on the D4-branes is

$$ds_6^2 = \left(\frac{u}{R_{AdS}}\right)^2 (-dt^2 + \delta_{ij} dx^i dx^j) + \left(\frac{u}{R_{AdS}}\right)^2 \left(f(u) + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}\right) dx_4^2 \tag{3.8}$$

Substituting the determinant of the metric and the pullback of the RR 5-form potential

$C_{(5)}$ into the action (3.7),we get

$$S_{D4} = \hat{T}_4 e^{-\phi} \int dx_4 \left(\frac{u}{R_{AdS}}\right)^5 \left[\sqrt{f(u) + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}} - a\right] \tag{3.9}$$

where $\hat{T}_4$ includes the outcome integration over all coordinates apart from dx_4 and $a \equiv \frac{2}{\sqrt{5}}\tilde{a}$.

The action does not depends explicitly on x_4 therefore the Hamiltonian will be conserved:

$$\left(\frac{u}{R_{AdS}}\right)^5 \left(\frac{f(u)}{\sqrt{f(u) + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}}} - a\right) = \left(\frac{u_0}{R_{AdS}}\right)^5 (\sqrt{f(u_0)} - a) \tag{3.10}$$

where u_0 is a point of a vanishing profile $u'(u)\big|_{u_0} = 0$.

Defining $y \equiv \frac{u}{u_0}$, $y_\Lambda = \frac{u_\Lambda}{u_0}$, $f(y) \equiv 1 - \left(\frac{y_\Lambda}{y}\right)^5$, the profile reads

$$u' = \left(\frac{u}{R_{AdS}}\right)^2 f(y) \sqrt{\frac{f(y)}{(y^{-5}\sqrt{f(1)} + a(1-y^{-5}))^2} - 1} \tag{3.11}$$

At $u \to \infty$ we want N_f D4-branes to be localized at $x_4 = 0$ and N_f $\overline{D4}$-branes at $x_4 = L$. These branes can't go to the interior of the space because they don't have where to end inside the "cigar". Therefore they should smoothly connect at some point $u = u_0$ ($u_0 \leq u_\Lambda$) and therefore at zero temperature chiral symmetry is broken.

We can express L as a function of u_0, u_Λ and R_{AdS}:

$$\begin{aligned} L &= \int_0^L dx_4 = 2\int_{u_0}^{\infty} \frac{du}{u'} \\ &= 2u_0 \int_1^{\infty} dy \left(\frac{R_{AdS}}{u}\right)^2 \frac{1}{f(y)} \frac{1}{\sqrt{\frac{f(y)}{(y^{-5}\sqrt{f(1)} + a(1-y^{-5}))^2} - 1}} \end{aligned} \tag{3.12}$$

Setting $z \equiv y^{-5}$, we get

$$L = \frac{2R_{AdS}^2}{5u_0} \int_0^1 dz \frac{1}{z^{\frac{4}{5}}(1 - y_\Lambda^5 z)} \frac{z(1-y_\Lambda^5) + a(1-z)}{\sqrt{1 - y_\Lambda^5 z - (z(1-y_\Lambda^5) + a(1-z))^2}} \tag{3.13}$$

From here we see that small values of L correspond to large values of u_0 and to $y_\Lambda << 1$.

In this limit $L \propto \frac{R_{AdS}^2}{u_0}$. The general dependence of L on u_0 is more complicated.

4 Near extremal AdS_6 model with flavor $D4$-$\overline{D4}$ branes at finite temperature

4.1 Bulk thermodynamics

We consider flavor branes as probes with $N_f \ll N_c$ and therefore we can analyze the thermodynamics of our model at finite temperature by considering only background geometry and then add probe D4-branes to the dominant bulk background at each temperature. In the gravity approximation (large N_c limit) we should look at Euclidean backgrounds, which are asymptotically (3.4), but with Euclidean and periodic time with a periodicity $t = 1/T = \beta$ and with anti-periodic boundary conditions for the fermions along the time direction in addition to the x_4 direction. This background is just a Euclidian continuation of the background (3.4) with the x_4 compact direction with a periodicity $2\pi R$ (with R related to u_Λ by (3.5)) that shrinks to zero at $u = u_0$ and with the time direction that remains always finite with an arbitrary periodicity equal to β (see figure 1(a)).

But now we can consider another solution with the same asymptotics, which is given by exchanging the behavior of the t and x_4 circles (i.e. by moving $f(u)$ in the metric (3.4) from the dx_4^2 term to the dt^2 term). Then now the time direction shrinks to zero size at $u = u_T$ (u_T is related to β), while the x_4 circle never shrinks (see figure 1(b)).

4.2 The bulk free energies of the low and high temperature phases

In order to decide which background dominates at a given temperature we need to compute their free energies. We look at the difference between the free energies, which is proportional to the difference between the actions of the backgrounds times the temperature (in the gravitational approximation), because it turns out to be finite despite that classical actions might diverge. In our calculations we use the notations and the results for the action computed in [6].

The class of Euclidean metrics that we are looking on can be parameterized as

$$l_s^{-2} ds^2 = d\tau^2 + e^{2\lambda(\tau)} dx_{||}^2 + e^{2\tilde{\lambda}(\tau)} dx_c^2 \tag{4.14}$$

where x_c is either x_4 or t (the one whose circle shrinks to zero size at the minimal value of u at a certain temperature), $x_{||}$ are the other four coordinates of $R^{4,1}$ (one of which is also compactified), τ is the radial direction and

$$e^{2\lambda} = \left(\frac{u}{R_{AdS}}\right)^2 \qquad e^{2\tilde{\lambda}} = \left(\frac{u}{R_{AdS}}\right)^2 \left(1 - \left(\frac{u_\Lambda}{u}\right)^5\right) \tag{4.15}$$

The functions λ and $\tilde{\lambda}$ depend only on the radial coordinate. The color D4-brane wrap the circle x_c, while the flavor D4-brane are points on the circle. The background also includes a constant dilaton ϕ_0 and a 6-form RR field strength. We define a deformed dilaton φ and a new radial coordinate ρ as

$$\begin{aligned} &\varphi = 2\phi_0 - 4\lambda - \tilde{\lambda} \\ &d\tau = -e^{-\varphi} d\rho \end{aligned} \tag{4.16}$$

Since the background depends only on a radial direction, the sugra action reduces to the following (0+1)-dimensional action:

$$\begin{aligned} S &= V \int d\rho \big(-4(\lambda')^2 - (\tilde{\lambda}')^2 + (\varphi')^2 + 4e^{-2\varphi} - Q_c^2 e^{4\lambda + \tilde{\lambda} - \varphi} \big) \\ &= -V \int_{u_\Lambda}^{\infty} du \left[\big(-4\dot{\lambda}^2 - \dot{\tilde{\lambda}}^2 + \dot{\varphi}^2 \big) \frac{du}{d\rho} + \left(4e^{-4\phi_0} - Q_c^2 e^{-2\phi_0} \right) e^{8\lambda + 2\tilde{\lambda}} \frac{d\rho}{du} \right] \end{aligned} \tag{4.17}$$

where V is the volume of all other directions except ρ in string units and Q_c is a constant that corresponds to the contribution of the RR flux. After rewriting the action in terms of integrals over u the minus sign arises because $d\rho/du$ is negative (dots denote derivatives with respect to u). Here we wrote the solution of the low temperature phase, for the high temperature phase one should replace u_Λ with u_T.

The equations of motion associated with the action (4.17) are

$$\begin{aligned} &\lambda'' - \frac{1}{2} Q_c^2 e^{2(4\lambda + \tilde{\lambda} - \phi_0)} = 0 \\ &\tilde{\lambda}'' - \frac{1}{2} Q_c^2 e^{2(4\lambda + \tilde{\lambda} - \phi_0)} = 0 \end{aligned} \tag{4.18}$$

The most general solution of this system is ([6]):

$$\lambda = -\frac{1}{5}\ln(\sinh(-5b\rho)) + 4b\rho \tag{4.19}$$
$$\tilde{\lambda} = -\frac{1}{5}\ln(\sinh(-5b\rho)) - b\rho$$

where $b = -\frac{1}{\sqrt{10}}Q_c e^{-\phi_0}$.

Substituting (4.15) into (4.17) we get

$$S = V\int_{u_\Lambda}^{\infty} du\Bigg[\left(\frac{20}{u^2}\frac{1}{1-\left(\frac{u_\Lambda}{u}\right)^5}\right)\frac{du}{d\rho}$$
$$+\left\{\left(4e^{-4\phi_0} - Q_c^2 e^{-2\phi_0}\right)\left(\frac{u}{R_{AdS}}\right)^{10}\left(1-\left(\frac{u_\Lambda}{u}\right)^5\right)\right\}\frac{d\rho}{du}\Bigg] \tag{4.20}$$

Using the solutions of the equations of motion associated with the above action (4.19) and (4.15)

$$\tilde{\lambda} - \lambda = 5b\rho \tag{4.21}$$
$$\frac{e^{2\tilde{\lambda}}}{e^{2\lambda}} = 1 - \left(\frac{u_\Lambda}{u}\right)^5 = e^{10b\rho}$$

we find that

$$\frac{d\rho}{du} = \frac{1}{2bu}\left(\frac{u_\Lambda}{u}\right)^5\frac{1}{1-\left(\frac{u_\Lambda}{u}\right)^5} \tag{4.22}$$

Substituting the expression into the action we find that the divergence at large u is independent of u_Λ, so it makes sense to subtract the expressions with u_Λ and with u_T to obtain a finite answer. The result for the difference between the action densities in the low temperature phase and in the high temperature phase is given by (defining $\hat{b} = b/u_\Lambda^5$ which is constant independent of u_Λ)

$$\frac{\Delta S}{V_3} \equiv \frac{S_{low} - S_{high}}{V_3} = \frac{2\pi R\beta}{l_s^2}\left(2\hat{b} + \left(4e^{-2\phi_0} - Q_c^2\right)e^{-2\phi_0}\frac{1}{10\hat{b}R_{AdS}^{10}}\right)(u_T^5 - u_\Lambda^5) \tag{4.23}$$

Using (3.5) and (4.31) we find

$$u_\Lambda = \frac{2R_{AdS}^2}{5R} \quad \text{and} \quad u_T = \frac{4\pi R_{AdS}^2}{5\beta} \tag{4.24}$$

Therefore the action is proportional to

$$\Delta S \propto N_c^2 \left(\frac{1}{(\beta/2\pi)^5} - \frac{1}{R^5} \right) \tag{4.25}$$

This result should be compared to the result derive in the critical model [1] In that case the power of β and R in the denominators were found to be six. A dependence on the fifth power as we found here seems to be more adequate for a dual five dimensional field theory. Both backgrounds have equal free energy when both circles are equal, i.e. $\beta = 2\pi R$. When the temperature is less than $T_d = 1/2\pi R$ the background in which x_4 circle shrinks to zero size dominates and when temperature is greater than $T_d = 1/2\pi R$ the background with t circle shrinking to zero dominates. There is a phase transition of first order here since two different configurations are possible at the transition point. If we compute the quark-antiquark potential (using the methods of [17–19]) , which is proportional to $\sqrt{g_{tt}g_{xx}}$, in the two backgrounds, we find that in the low-temperature background it is finite at u_0 and linear, corresponding to a confined phase, and in the high-temperature it decays, corresponding to a deconfined phase.

4.3 Low temperature phase

As described above the background corresponding to the low temperature phase is the one with the x_4 circle shrinking to zero at $u = u_0$. The only difference from the zero temperature case is that the time direction is Euclidean and compactified with a circumference $\beta = 1/T$. Hence, at low temperatures the dual gauge theory is in the confining phase. When we add flavor branes and anti-brains to the background they have no other possibility than to connect because x_4 shrinks to zero size. Therefore chiral symmetry is broken at least until the temperatures corresponding to deconfinement.

The metric is

$$\begin{aligned}
&ds_6^2 = \left(\frac{u}{R_{AdS}} \right)^2 (dt^2 + \delta_{ij} dx^i dx^j + f(u) dx_4^2) + \left(\frac{R_{AdS}}{u} \right)^2 \frac{du^2}{f(u)} \\
&f(u) = 1 - \left(\frac{u_\Lambda}{u} \right)^5 \\
&x_4 \sim x_4 + 2\pi R = x_4 + \frac{4\pi R_{AdS}^2}{5u_\Lambda} \quad \text{and} \quad t \sim t + \beta \ (\beta \text{ arbitrary})
\end{aligned} \tag{4.26}$$

In the next section we will see that we can make sense of a holographic duality only for

the case of chargeless branes with vanishing $\tilde{a}$ Setting $\tilde{a}$ to zero, we find from (3.11)

$$u' = \left(\frac{u}{R_{AdS}}\right)^2 f(y)\sqrt{y^{10}\frac{f(y)}{f(1)} - 1} \tag{4.27}$$

Substituting (4.27) into (3.7) and using the definitions $y \equiv \frac{u}{u_0}$, $y_\Lambda = \frac{u_\Lambda}{u_0}$, $f(y) \equiv 1 - \left(\frac{y_\Lambda}{y}\right)^5$ and $z \equiv y^{-5}$, we get the following DBI action:

$$\begin{aligned} S_{DBI} &= \hat{T}_4 e^{-\phi}\int dx_4 \left(\frac{u}{R_{AdS}}\right)^5 \sqrt{f(u) + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}} \\ &= \frac{2\hat{T}_4 e^{-\phi} u_0^4}{5R_{AdS}^3}\int_0^1 dz \frac{1}{z^{\frac{9}{5}}}\frac{1}{\sqrt{1 - y_\Lambda^5 z - z^2(1-y_\Lambda^5)}} \end{aligned} \tag{4.28}$$

The relation between L and u_0 is the same as at zero temperatures. For small L the dependence of the action on L is:

$$S_{DBI} \propto \frac{\hat{T}_4 e^{-\phi}}{R_{AdS}^3 L^4} \tag{4.29}$$

4.4 Intermediate and high temperature phases

In the high temperature phase the background metric takes the form

$$\begin{aligned} ds_6^2 &= \left(\frac{u}{R_{AdS}}\right)^2 (f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)} \\ f(u) &= 1 - \left(\frac{u_T}{u}\right)^5 \end{aligned} \tag{4.30}$$

Now the time circle shrinks to zero at the minimal value of $u = u_T$ and to avoid a singularity there the time direction should be identified with the periodicity

$$t \sim t + \beta = t + \frac{4\pi R_{AdS}^2}{5u_T} \tag{4.31}$$

On the other hand the periodicity of x_4 is now arbitrary:

$$x_4 \sim x_4 + 2\pi R \tag{4.32}$$

D4-branes span the same coordinates as previously and are described by some profile $u(x_4)$. The induced metric and the DBI action now takes the form

$$ds_6^2 = \left(\frac{u}{R_{AdS}}\right)^2 (f(u)dt^2 + \delta_{ij}dx^i dx^j) + \left(\frac{u}{R_{AdS}}\right)^2 \left(1 + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}\right) dx_4^2 \tag{4.33}$$

We get the following action:

$$\begin{aligned}
S_{D4} &= T_4 \int d^5x e^{-\phi} \sqrt{-det\hat{g}} - \frac{\sqrt{5}a}{2} T_4 \int \mathcal{P}(C_{(5)}) \\
&= \hat{T}_4 e^{-\phi} \int dx_4 \left(\frac{u}{R_{AdS}}\right)^5 \left[\sqrt{f(u)} \sqrt{1 + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}} - a\right] \\
&= 2\hat{T}_4 e^{-\phi} \left[\int_{u_0}^{\infty} \frac{du}{u'} \left(\frac{u}{R_{AdS}}\right)^5 \sqrt{f(u)} \sqrt{1 + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}}\right. \\
&\quad \left. - a \int_{u_0}^{\infty} \frac{du}{u'} \left(\frac{u}{R_{AdS}}\right)^5\right]
\end{aligned} \tag{4.34}$$

where here we turned on the CS term and took non-vanishing $a \leq 1$. Conservation of the Hamiltonian of (4.34) implies that

$$\left(\frac{u}{R_{AdS}}\right)^5 \left(\frac{\sqrt{f(u)}}{\sqrt{1 + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}}} - a\right) = const \tag{4.35}$$

There is a solution for a vanishing profile at some point $u_0 \geq u_T$, where $u'(u)\big|_{u_0} = 0$ (see figures 2(a) and 2(b)). This is a solution for branes and anti-branes connected at $u = u_0$, where u' should be zero. At low temperatures this was the only possible configuration, but since in the high temperature phase the x_4 circle never shrinks to zero size we can consider a configuration of non-intersecting branes and antibranes that end on the horizon of the black hole (see figure 2(c)). The branes and antibranes stay disconnected in the $u - x_4$ submanifold with constant values $x_4(u) = 0, L$, e.g chiral symmetry is restored. Since the branes are now parallel to each other $u' = \infty$. Eventhough this is not a solution of (4.35) it is a solution of the equation of motion associated with the action (4.34) [1].

Inspite of the fact that the both the parallel and the U shape solutions exist for non trivial a we found that for that case (as is desribed in the appendix) the difference of the free energies diverges. Thus we switch off again the CS term. Defining $y \equiv \frac{u}{u_0}$, $y_T = \frac{u_T}{u_0}$,

[1] We thank O. Aharoni for pointing this to us

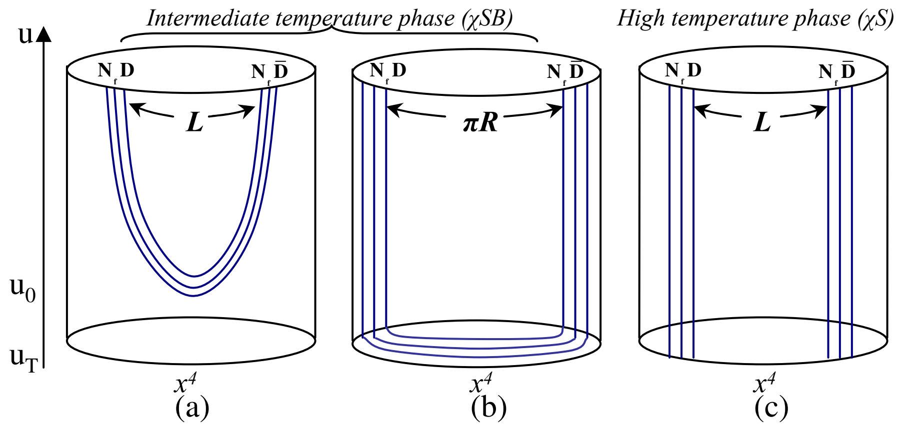

Figure 2: The three possible configurations of the flavor D4-branes and antibranes at intermediate and high temperatures: (a) a configuration of connected branes at the minimum $u = u_0$ with an asymptotic separation L at $u \to \infty$ (intermediate temperature phase), (b) a configuration of connected branes in the case $u_0 = u_T$ (intermediate temperature phase), (c) parallel branes configuration (high temperature phase).

$f(y) \equiv 1 - \left(\frac{y_T}{y}\right)^5$, the profile velocity now reads

$$u' = \left(\frac{u}{R_{AdS}}\right)^2 \sqrt{f(y)} \sqrt{y^{10}\frac{f(y)}{f(1)} - 1} \tag{4.36}$$

Then x_4 as a function of u becomes

$$x_4(u) = \int_{u_0}^{u} d\hat{u} \frac{1}{\hat{u}'} = \int_{u_0}^{u} d\hat{u} \frac{1}{\left(\frac{\hat{u}}{R_{AdS}}\right)^2 \sqrt{1 - (\frac{u_T}{\hat{u}})^5} \sqrt{\left(\frac{\hat{u}}{u_0}\right)^5 \frac{1-(\frac{u_T}{\hat{u}})^5}{1-u_T^5} - 1}} \tag{4.37}$$

It is shown in figure 4. At the beginning when $u \approx u_0$ the profile of the branes growth very rapidly and when $u \to \infty$ the profile is almost straight. For $u_0 \approx u_T$ the profile is drawn at figure 2(b).

By substituting the outcome of the equation of motion into the action, we get

$$S_{DBI}^{high} = \frac{2\hat{T}_4 e^{-\phi} u_0^4}{R_{AdS}^3} \int_1^{\infty} dy \frac{y^3}{\sqrt{1 - \frac{f(1)}{f(y)y^{10}}}} \tag{4.38}$$

The velocity of the profile of a parallel branes configuration is always $u' \to \infty$ and

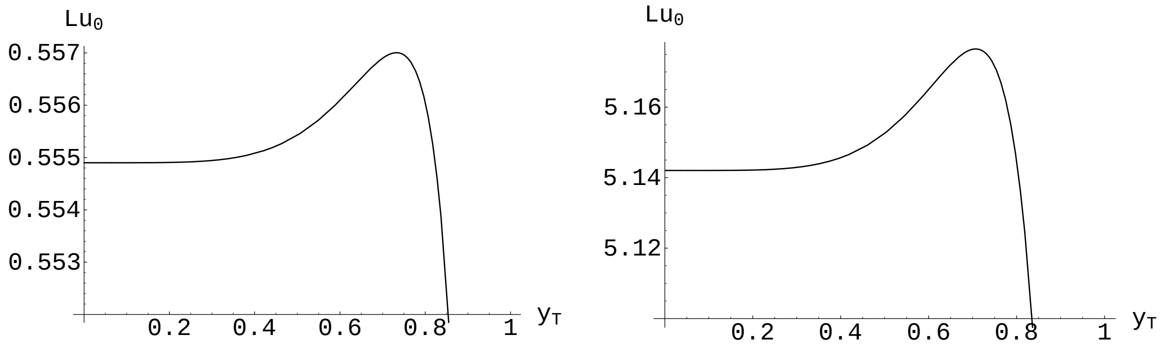

Figure 3: Lu_0 as a function of y_T. The left graph is for the case $\tilde{a} = 0$ and the right one is for the case $\tilde{a} = 1$.

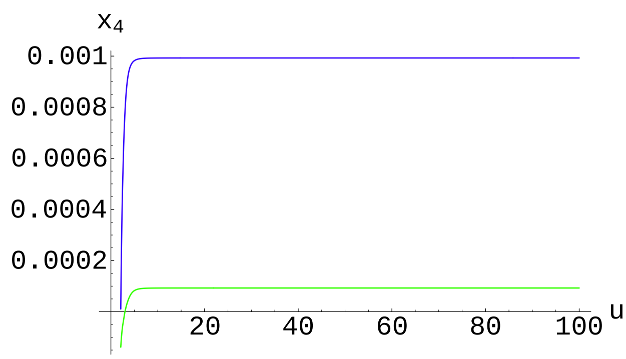

Figure 4: Profile x_4 as a function of u. The lower curve (green) is for $u_T = 2$, $u_0 = 3$. The upper curve (blue) is for the case $u_0 \approx u_T = 2$ ($R_{AdS} = 1$).

therefore the action reads

$$
\begin{aligned}
S^{high}_{DBI\,u'\to\infty} &= 2\hat{T}_4 e^{-\phi}\int_{u_T}^{\infty} du \left(\frac{u}{R_{AdS}}\right)^5 \left(\frac{R_{AdS}}{u}\right)^2 \\
&= \frac{2\hat{T}_4 e^{-\phi} u_0^4}{R_{AdS}^3}\left[\int_1^{\infty} dy y^3 + \int_{y_T}^{1} dy y^3\right]
\end{aligned} \tag{4.39}
$$

To find whether a configuration with χSB or with a restored χS is preferred we can compute the difference between the actions of the two configuration that is proportional

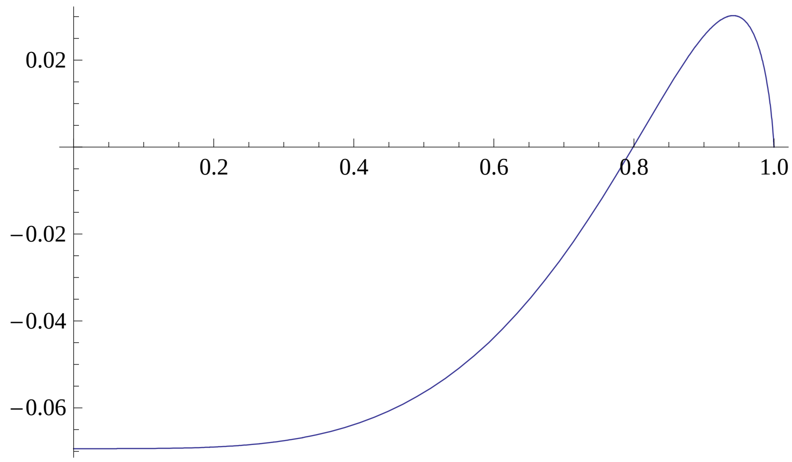

Figure 5: ΔS as a function of y_T, in units of $2\hat{T}_4 e^{-\phi} u_0^4 / R_{AdS}^3$ in the case $\tilde{a} = 0$.

to free energy. Configuration that has a lower free energy is preferred.

$$\begin{aligned}
\Delta S &\equiv \frac{R_{AdS}^3}{2\hat{T}_4 e^{-\phi} u_0^4} (S_{DBI}^{high} - S_{DBI u' \to \infty}^{high}) \\
&= \int_1^\infty dy y^3 \left[\frac{1}{\sqrt{1 - \frac{f(1)}{f(y) y^{10}}}} - 1 \right] - \int_{y_T}^1 dy y^3 \\
&= \frac{1}{5} \int_0^1 dz \frac{1}{z^{\frac{9}{5}}} \left[\sqrt{\frac{1 - y_T^5 z}{1 - y_T^5 z - z^2 (1 - y_T^5)}} - 1 \right] - \frac{1}{4} (1 - y_T^4)
\end{aligned} \tag{4.40}$$

where was introduced $z = y^{-5}$ change of variables. ΔS as a function of y_T is drawn in figure 5. When $y_T > 0.8$ ΔS is positive, i.e. $S_{DBI u' \to \infty}^{high}$ has a lower free energy and is preferred. In this phase D4-brane are disconnected and chiral symmetry is restored, while when $0 < y_T < 0.8$ D4-branes are smoothly connected and chiral symmetry is broken.

We would like to express the critical point in terms of physical quantities. For a certain value of y_T we can compute an integral that relates the minimal point of the connected

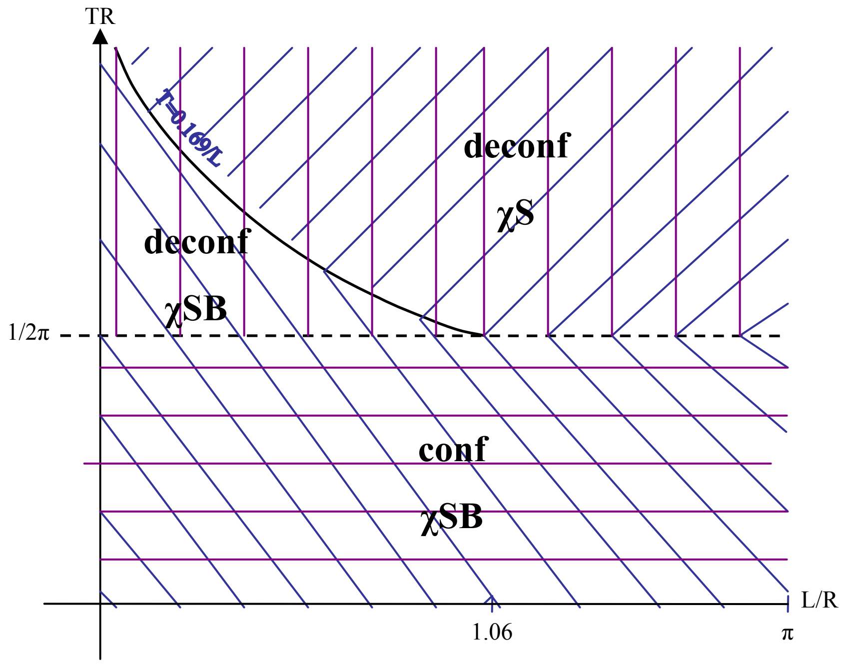

Figure 6: The phase diagram of the AdS_6 model with flavor D4-branes. The phase structure depends only on the two dimensionless parameters TR and L/R. For $L/R < 1.06$ the deconfinement and chiral symmetry restoration transitions happens at different temperatures, while for $L/R > 1.06$ they occur together .

branes configuration u_0 to the asymptotic distance between branes and antibranes L.

$$L = 2\int_{u_0}^{\infty} \frac{du}{u'} = \frac{2R_{AdS}^2}{u_0}\int_1^{\infty} dy \frac{1}{y^2\sqrt{f(y)}}\frac{1}{\sqrt{y^{10}\frac{f(y)}{f(1)} - 1}}$$
$$= \frac{2R_{AdS}^2}{5u_0}\sqrt{1-y_T^5}\int_0^1 dz \frac{1}{\sqrt{1-y_T^5 z}}\frac{z^{\frac{1}{5}}}{\sqrt{1-y_T^5 z - z^2(1-y_T^5)}} \tag{4.41}$$

For small values of L $u_0 \propto R_{AdS}^2/L$. At the transition temperature $y_T^c = 0.8$ the integral (4.41) gives $L = 0.53(R_{AdS}^2/u_0)$. From the equation (4.31) we find

$$T_{\chi SB} = \frac{5y_T u_0}{4\pi R_{AdS}^2} = \frac{5y_T 0.53}{4\pi L} = \frac{0.169}{L} \tag{4.42}$$

while the deconfinement phase transition happens at the temperature

$$T_d = \frac{1}{2\pi R} = \frac{0.159}{R} \tag{4.43}$$

Both temperatures are equal when $L = 1.06R$. For $L/R > 1.06$ and $T \cdot R > T_d \cdot R$ the system is deconfined and chiral symmetry is restored, while for $L/R < 1.06$ and temperatures bigger than the temperature of deconfinement the system is deconfined but chiral symmetry restoration happens separately: at $T \cdot R < T_{\chi SB} \cdot R$ chiral symmetry is still broken and at $T \cdot R > T_{\chi SB} \cdot R$ chiral symmetry is restored. The full phase diagram of the theory is drawn in figure 6.

4.5 General model

From the similarity of the result of the previous section to the SS model we can derive a general model, but it is not necessary that all three phases will be present. We indeed find a different behavior of some metrics.

We consider a n-dimensional Wick rotated black hole background and insert into it (n-2)-probe branes, that extend along all directions except x_4. We take the following general form of the metric at low temperatures:

$$ds_n^2 = H_2 dt^2 + \frac{1}{H_1} du^2 + H_1 dx_4^2 + ds_k^2 \tag{4.44}$$

and then the metric at high temperatures becomes:

$$ds_n^2 = H_1 dt^2 + \frac{1}{H_1} du^2 + H_2 dx_4^2 + ds_k^2 \tag{4.45}$$

where H_1 is a singular function of u with a horizon at $u = u_H$, H_2 is a non-singular function of u, x_4 is compact with a period that depends on u_H and ds_k^2 is any k-dimensional metric of the rest of coordinates, whose components can depend on u. The condition of the singular H_1 is necessary to have a horizon on which (n-2)-branes can end. At low temperature the $u - x_4$ submanifold is cigar-shaped, while at high temperature the $u - t$ submanifold is cigar-shaped and the x_4 circle does not shrink to zero.

From (4.45) we find the induced metric on the (n-2)-probe brane

$$ds_n^2 = H_1 dt^2 + (H_2 + \frac{1}{H_1} u'^2) dx_4^2 + ds_k^2 \tag{4.46}$$

with $u' = du/dx_4$.

The DBI action is given by (without the CS term)

$$S_{DBI} = T_n \int d^{n-1}x e^{-\phi}\sqrt{g}\sqrt{H_1(H_2 + \frac{1}{H_1}u'^2)} = \hat{T}_n \int dx_4 e^{-\phi}\sqrt{g}\sqrt{H_1(H_2 + \frac{1}{H_1}u'^2)} \tag{4.47}$$

where $\hat{T}_n$ includes the outcome integration over all coordinates apart from dx_4, e^{ϕ} is a dilaton and g is the determinant of the ds_k^2 metric. Then from the conservation of the Hamiltonian we find that the equation of motion is

$$\frac{e^{-\phi}\sqrt{g}\sqrt{H_1}H_2}{\sqrt{H_2 + \frac{1}{H_1}u'^2}} = const \tag{4.48}$$

If we suppose that there is a solution with $u'(u = u_0) = 0$, then the constant is equal to $e^{-\phi_0}\sqrt{g^0 H_1^0 H_2^0}$ and

$$u' = \sqrt{H_1 H_2}\sqrt{\frac{e^{-2\phi} g H_1 H_2}{e^{-2\phi_0} g^0 H_1^0 H_2^0} - 1} \tag{4.49}$$

The case $u'(u = 0) \to \infty$ is also a solution of the equation of motion and gives $const = 0$.

Inserting the expression for u' into (4.47) we find

$$S_{DBI} = 2\hat{T}_n \int_{u_0}^{\infty} du e^{-\phi}\sqrt{g}\frac{1}{\sqrt{1 - \frac{e^{-2\phi_0} g^0 H_1^0 H_2^0}{e^{-2\phi} g H_1 H_2}}} \tag{4.50}$$

At $u' \to \infty$ the actions reads

$$S_{DBI}^{u' \to \infty} = 2\hat{T}_n \int_{u_0}^{\infty} du e^{-\phi}\sqrt{g} \tag{4.51}$$

The action of the parallel brane configuration actually depends only on the x^k coordinates and the dilaton. The difference between the actions is

$$\Delta S = 2\hat{T}_n \int_{u_0}^{\infty} du e^{-\phi}\sqrt{g}\left[\frac{1}{\sqrt{1 - \frac{e^{-2\phi_0} g^0 H_1^0 H_2^0}{e^{-2\phi} g H_1 H_2}}} - 1\right] - \int_{u_T}^{u_0} du e^{-\phi}\sqrt{g} \tag{4.52}$$

To evaluate it we need to insert the explicit expressions of the functions H_1, H_2, g and the dilaton.

The existence of the solution with $u'(u = u_0) = 0$ and $u_0 \neq 0$ at low temperatures

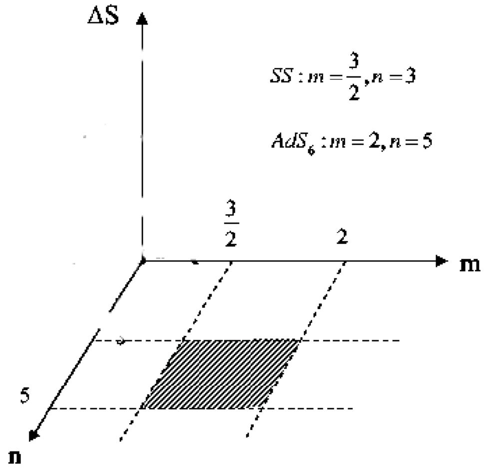

Figure 7: Models with values of n between 3 and 5, m between 3/2 and 2, and arbitrary j and l should have intermediate and high temperature phases.

guarantees confinement, but to see whether we get chiral symmetry restoration or not we need an explicit form of the functions ϕ, g, H_1 and H_2. Let us look at the same family of metrics to which SS and AdS_6 metrics belong. That is:

$$\begin{aligned}
H_1 &= \left(\frac{u}{R}\right)^m \left(1 - \left(\frac{u_T}{u}\right)^n\right) \\
H_2 &= \left(\frac{u}{R}\right)^m \\
e^{-\phi} &= const_1 \cdot u^l \\
g &= const_2 \cdot u^j
\end{aligned} \tag{4.53}$$

$const_1$ and $const_1$ does not effect δS and therefore we them arbitrary. Substituting the functions (4.53) into the action's difference (4.52) and defining $y = u/u_0$, we get:

$$\Delta S \propto \int_1^\infty dy y^{l+\frac{j}{2}} \left(\frac{1}{\sqrt{1 - y^{-(2l+j+2m)} \frac{1-y_T^n}{1-(y_T/y)^n}}} - 1 \right) - \int_{y_T}^1 dy y^{l+\frac{j}{2}} \tag{4.54}$$

We did the numerical computations for different values of j, l, m, n (since we cannot solve the first integral analytically). We checked that for n between 3 and 5, for m between 3/2 and 2, and arbitrary j and l ΔS is negative and then positive as in the SS

and AdS_6 models, as expected (see figure 7).

Also we can see whether we get a different phase structure for general critical and non-critical versions of near extremal Dp-branes. The metric for the critical near extremal Dp-branes is [20]:

$$
\begin{aligned}
ds^2 &= \frac{u^{(7-p)/2}}{R_{Dp}^2}\left(-\left(1-\frac{u_T^{7-p}}{u^{7-p}}\right)dt^2 + dx_4^2\right) + \frac{R_{Dp}^2}{u^{(7-p)/2}}\frac{1}{1-\frac{u_T^{7-p}}{u^{7-p}}}du^2 \\
&+ \frac{u^{(7-p)/2}}{R_{Dp}^2}\delta_{ij}dx^i dx^j + R_{Dp}^2 u^{(p-3)/2} d\Omega_{8-p}^2 \qquad i,j = 1,...,p-1 \\
e^{-\phi} &= \frac{1}{(2\pi)^{2-p} g_{YM}^2 R_{Dp}^{3-p}} u^{(7-p)(3-p)/4} \qquad R_{Dp}^2 = g_{YM}\sqrt{N}
\end{aligned}
\tag{4.55}
$$

Therefore all the powers depend only on p and we find:

$$
\begin{aligned}
m &= \frac{7-p}{2} \\
n &= 7-p \\
l &= \frac{(7-p)(3-p)}{4} \\
j &= \frac{(7-p)(p-1)+(p-3)}{2}
\end{aligned}
\tag{4.56}
$$

Since the solutions are in 10 dimensions, we insert D8-probe branes because if there are directions along which probe branes do not extend, except the x_4 direction, the massive quarks appear and we will not get chiral symmetry. Drawing the ΔS (4.54) numerically for different values of p we find that for $p \leq 5$ we get the same behavior as in the SS model (two phases - chiral symmetry breaking and restoration), but for $p = 6$ we get a positive ΔS that means that chiral symmetry restoration and deconfinement occur together (see figure 8). For $p > 6$ we cannot get the solution of (4.54) numerically.

Near extremal solutions of Dp-branes in non-critical dimensions are given by [6]:

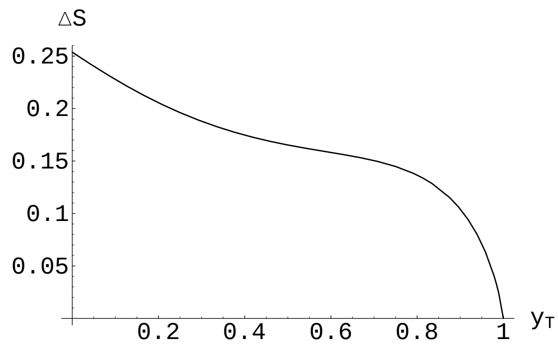

Figure 8: ΔS as a function of y_T for the near extremal D6-branes in 10 dimensions.

$$
\begin{aligned}
ds^2 &= \left(\frac{u}{R_{AdS}}\right)^2 \left(-\left(1-\frac{u_T^{p+1}}{u^{p+1}}\right)dt^2 + dx_4^2\right) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{1}{1-\frac{u_T^{p+1}}{u^{p+1}}} du^2 \\
&+ \left(\frac{u}{R_{AdS}}\right)^2 \delta_{ij}dx^i dx^j + R_{S^q}^2 d\Omega_q^2 \qquad i,j \text{ arbitrary but smaller than } 7-q \\
e^{-\phi_0} &= \left[\frac{1}{p+2-q}\left(\frac{(p+2-q)(q-1)}{c}\right)^q \frac{2c}{Q^2}\right]^{-1/2} \qquad R_{AdS}^2 = \frac{(p+1)(p+2-q)}{c}
\end{aligned}
\tag{4.57}
$$

where

$$\frac{c}{\alpha'} = \frac{10-d}{\alpha'} \tag{4.58}$$

is the non-criticality central charge term.

We see that again all the powers depend only on dimension of the branes p:

$$
\begin{aligned}
m &= 2 \\
n &= p+1 \\
l &= 0 \\
j &= 6
\end{aligned}
\tag{4.59}
$$

The probe branes that we insert into the backgrounds are (d-2)-branes and antibranes (d - is the dimension of a non-critical metric). For different values of $n = p+1$ we find that

ΔS (4.54) always has the same behavior as in the AdS_6 BH model, i.e. chiral symmetry can be restored at a higher temperature then the temperature of deconfinement, but the value of the ratio L/R will be different for different Dp-branes.

From the above analysis we see that the phase structure of a model depends on the basic structure and dimensionality of its metric and it can be different for different models.

5 Spectrum of mesons

The mesons of our model are described by strings ending on the probe D4-branes. Low-spin mesons are described via modes of the massless fields living on the D4-branes and high-spin mesons are associated with string configurations that fall from the D4-branes down to the wall at $u = u_\Lambda$, stretch along the wall and then go back up again. The mesonic spectrum in the low-temperature phase is unchanged as the temperature is increased because in the confining phase the theory behaves effectively as a gas of non-interacting glueballs and mesons [21,22]. However, the mesonic spectrum at intermediate temperature might be not connected to the spectrum in the low-temperature regime since the phase transition is first-order and such a jump should be expected. Now we turn our attention to the meson spectrum at the intermediate and high temperature phases.

5.1 Low-spin mesons at intermediate temperature

Low-spin mesons correspond on the string theory side to fluctuations of the massless fields on the probe branes. The fluctuations of the gauge fields on the branes give pseudo-vector and scalar mesons and pions, and the fluctuations of the scalar field describing the embedding of the branes give massive scalar mesons. Using the analysis of the fluctuations performed in [9] we describe the modes coming from the components of the gauge field living on the D4-branes.

The spectrum of low-spin mesons in the low-temperature phase is unmodified with respect to zero temperature since the Euclidean metric is globally unmodified.

The spectrum in the intermediate temperature phase is discrete because the probe does not intersect the horizon. Also computations in [4] for the SS model show that given that the effective tension of strings near the brane decreases with the increase of temperature, the masses of mesons decrease as the temperature is increased. This behavior is also true for our model.

We start from the background metric describing the hot gluonic plasma (4.30).The induced metric on the D4-brane worldvolume at intermediate temperature reads

$$\mathrm{d}s^2_{\text{interm}} = \left(\frac{u}{R_{AdS}}\right)^2 \left(f(u)\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) + \left[\left(\frac{R_{AdS}}{u}\right)^2 \frac{1}{f(u)} + \left(\frac{\mathrm{d}x^4}{\mathrm{d}u}\right)^2 \left(\frac{u}{R_{AdS}}\right)^2\right] \mathrm{d}u^2 \quad (5.60)$$

We are interested in computing the spectrum of vector mesons, by considering small fluctuations on the worldvolume gauge fields of the probe D4-brane. We expand the gauge field as [2]

$$A_\mu(x^\mu, u) = \sum_n B_\mu^{(n)}(x^\mu)\psi_{(n)}(u) \quad (5.61)$$

$$A_u(x^\mu, u) = \sum_n \varphi_\mu^{(n)}(x^\mu)\phi_{(n)}(u) \quad (5.62)$$

and therefore the field strength reads

$$\begin{aligned} F_{\mu\nu} &= \sum_n F_{\mu\nu}^{(n)}(x^\rho)\, \psi_n(u)\,, \\ F_{\mu u} &= \sum_n \partial_\mu\varphi^{(n)}\, \phi_n(u) - B_\mu^{(n)}\partial_u\psi_n(u) \\ &= \partial_\mu\varphi^{(0)}\, \phi_0 + \sum_{n\geq 1} \left(\partial_\mu\varphi^{(n)} - B_\mu^{(n)}\right) \partial_u\psi_{(n)}\,. \end{aligned} \quad (5.63)$$

where the last line is obtained by taking $\phi_{(n)} = m_n^{-1}\partial_u\psi_{(n)}(u)$. To simplify the consideration, we furthermore go to the $A_0 = 0$ gauge and consider only spatially homogeneous modes, i.e. we consider the equation of motion for fields satisfying $\partial_i A_j = 0$. Then the

probe brane action (with $a = 0$ is

$$\hat{S}_{\text{trunc}} = \int \mathrm{d}^4x\mathrm{d}u\; u^4\gamma^{1/2}\, f(u)^{1/2} \left[\frac{1}{u^2\gamma\, f(u)} (\partial_0\varphi^{(0)})^2\, \phi^{(0)}\phi^{(0)} \right.$$

$$\left. - \frac{1}{f(u)} \left(\frac{R_{AdS}}{u}\right)^4 \partial_0 B_i^{(m)} \partial_0 B^i_{(n)}\, \psi_{(m)}\psi_{(n)} + \frac{1}{u^2\gamma} B_i^{(m)} B^i_{(n)}\, \partial_u\psi_{(m)}\partial_u\psi_{(n)} \right],$$

$$\text{with} \qquad \gamma \equiv \frac{u^8}{u^{10}f(u) - u_0^{10}f(u_0)} \tag{5.64}$$

After a partial integration with respect to the u-coordinate, the equation of motion for the field $B_i^{(m)}$ becomes

$$\frac{u^2}{\gamma^{1/2}f(u)^{1/2}} \partial_0^2 B_i^{(n)} \psi_{(n)} - \partial_u \left(u^2\gamma^{-1/2} f(u)^{1/2}\; \partial_u\psi_{(n)}\right) B_i^{(n)} = 0 \tag{5.65}$$

This equation will reduce to the canonical form

$$\partial_0^2 B_i^{(n)} = -m_n^2 B_i^{(n)}\,, \tag{5.66}$$

if the modes $\psi_{(n)}$ satisfy the equation

$$-\,\gamma^{-1/2}\, f(u)^{1/2}\, \partial_u \left(u^2\gamma^{-1/2} f(u)^{1/2} \partial_u\psi_{(n)}\right) = R_{AdS}^4\, m_n^2\, \psi_{(n)}\,. \tag{5.67}$$

This equation is very similar to the equation in the zero temperature case computed in [9], the only difference is the appearance of the factor $f(u)^{1/2}$ in the term on the left-hand side. The modes should also satisfy the normalization conditions

$$\begin{aligned} &\int_{u_0}^{\infty} \mathrm{d}u\, \gamma^{1/2} f(u)^{-1/2}\, \psi_{(m)}\psi_{(n)} = \delta_{mn}\,, \\ &\int_{u_0}^{\infty} \mathrm{d}u\, \frac{u^2}{R_{AdS}^4} \gamma^{-1/2} f(u)^{-1/2}\, \phi^{(0)}\phi^{(0)} = 1\,. \end{aligned} \tag{5.68}$$

The zero mode $\phi^{(0)} = u^{-2} f(u)^{-1/2}\gamma^{1/2}$ is normalizable with this norm (there is no problem at the horizon because $u_0 > u_T$), and therefore there is a massless pion $\pi^{(0)}$ present in the intermediate-temperature phase. The fields $\pi^{(0)}$ are the Goldstone bosons associated with the spontaneous breaking of the $U(N_f)_L \times U(N_f)_R$ global chiral symmetry to the diagonal $U(N_f)$.

In the limit of $u_0 \gg u_T$ the spectrum simplifies and one can easily determine the scale of the meson masses. In this limit, which corresponds to a small separation distance between the stacks of branes and anti-branes $L \ll R$, the thermal factor $f(u) \to 1$ and in particular also $f(u_0) \to 1$.Therefore,

$$\gamma \equiv \frac{u^8}{u^{10} f(u) - u_0^{10} f(u_0)} \to \frac{1}{u^2} \frac{1}{1 - y^{-10}} \tag{5.69}$$

where the dimensionless quantity $y \equiv u/u_0$. Then we can rewrite (5.67) in terms of y in the following form

$$-\gamma^{-1/2}(y)\partial_y \left(y^2 \gamma^{-1/2}(y) \partial_y \psi_{(n)} \right) = \frac{R_{AdS}^4}{u_0^2} m_n^2 \, \psi_{(n)} \, . \tag{5.70}$$

Now since the left-hand side is expressed in terms of the dimensionless quantity y, the right-hand side should also be dimensionless which implies that

$$m_n^2 \sim \frac{u_0^2}{R_{AdS}^4} \tag{5.71}$$

From (4.41) we know that $u_0 \sim 1/L$. Therefore the mass of “short” mesons scales as

$$M_{\text{meson}} \sim \frac{1}{L} \, . \tag{5.72}$$

The explicit mass spectrum of the vector mesons can be found by looking for normalizable eigenfunctions of (5.67) and using numerical methods (e.g. a shooting technique), but the qualitative behavior of the spectrum is that the masses of mesons decrease as temperature increases. This behavior is a direct consequence of the fact that the constituent quark mass is related to the distance of the tip of the probe brane to the horizon. If the distance is increased, a meson of the same spin will correspond to an excitation of the brane which is further away from the horizon and hence less affected by the temperature. This behavior is common for all gravitational backgrounds that contain a horizon.

5.2 Low-spin mesons at high temperature

In the high-temperature phase the profile of the left and right stacks of branes is characterized by $u' \equiv \mathrm{d}u/\mathrm{d}x^4 \to \infty$ and the induced metric on the probe branes and probe

anti-branes takes the form

$$
\mathrm{d}\hat{s}^2_{\text{high}} = \left(\frac{u}{R_{AdS}}\right)^2 \left[-f(u)\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right] + \left(\frac{R_{AdS}}{u}\right)^2 \frac{1}{f(u)}\mathrm{d}u^2 \tag{5.73}
$$

The differential equation for the modes is now

$$
-\ f(u)^{1/2}\,\partial_u\left(u^2 f(u)^{1/2}\partial_u\psi_{(n)}\right) = R^4_{AdS}\, m_n^2\, \psi_{(n)} \tag{5.74}
$$

i.e. it is similar to the intermediate temperature phase case, but with $\gamma = 1$. Then the normalization conditions are now

$$
\begin{aligned}
&\int_{u_0}^{\infty} \mathrm{d}u\, f(u)^{-1/2}\,\psi_{(m)}\psi_{(n)} = \delta_{mn}\,, \\
&\int_{u_0}^{\infty} \mathrm{d}u\,\frac{u^2}{R^4_{AdS}} f(u)^{-1/2}\,\phi^{(0)}\phi^{(0)} = 1\,.
\end{aligned} \tag{5.75}
$$

The mode, which would be given by $\phi^{(0)} = u^{-2}f(u)^{-1/2}$, is no longer normalizable. Computation of its norm leads to the integral

$$
\int_{u_T}^{\infty} \mathrm{d}u\, u^2 f(u)^{-1/2}\Big|u^{-2}f(u)^{-1/2}\Big|^2\,, \tag{5.76}
$$

which, while convergent at the upper boundary, is divergent at the lower boundary because $f(u) \sim \sqrt{u - u_T}$ for $u \sim u_T$. In accordance with the fact that chiral symmetry is restored in the high-temperature phase, we see that the Goldstone boson has disappeared. In the high temperature phase the spectrum of vector mesons is continuous.

5.3 High-spin mesons at intermediate temperature

To describe higher-spin mesons we cannot use supergravity modes and we need to consider string configurations [23] that start and end on probe branes. For large spin these strings can be described semiclassically. The relevant string configuration can be decomposed into three parts: a segment from the probe brane at $u = u_0$ to the wall at $u = u_T$, then a segment that stretches along the wall in the spacial direction, and then another vertical part stretching from the wall back to the probe brane, as depicted at figure 9.

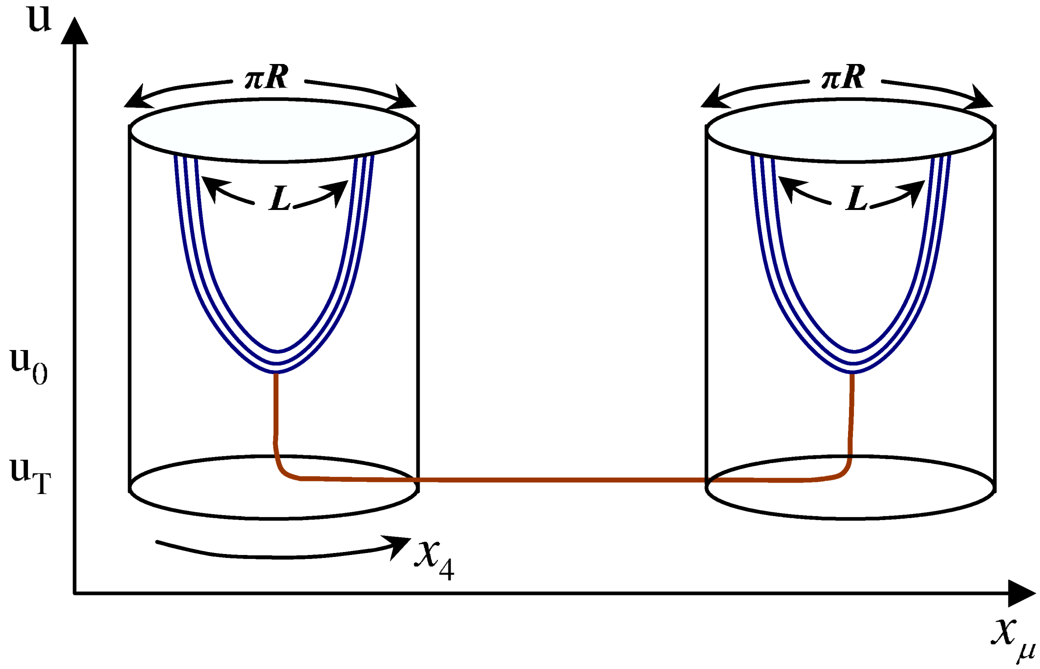

Figure 9: A high-spin meson at intermediate temperatures represented as a semiclassical string starting at the lowest point of the probe brane $u = u_0$, going down to the wall at $u = u_T$, stretching horizontally in the space along the wall, and then going back up vertically to the probe brane at $u = u_0$.

The relevant part of the background metric that represents this configuration is

$$\mathrm{d}s^2 = \left(\frac{u}{R_{AdS}}\right)^2 \left(-f(u)\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\varphi^2\right) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{\mathrm{d}u^2}{f(u)} \tag{5.77}$$

We go to the static gauge for the string action and make the following ansatz for the rotating configuration,

$$t = \tau\,, \quad \rho = \rho(\sigma)\,, \quad u = u(\sigma)\,, \quad \varphi = \omega\tau \tag{5.78}$$

This ansatz has the same form as in the zero-temperature case [23]. Hence, the only effect of finite temperature will be in the change of the shape of $u(\sigma)$ as the temperature is increased.

With this ansatz the metric now reads

$$\mathrm{d}s^2 = \left(\frac{u}{R_{AdS}}\right)^2 (-f(u) + \rho^2\omega^2)\mathrm{d}\tau^2 + \left(\frac{u}{R_{AdS}}\right)^2 \left(\rho'^2 + \left(\frac{R_{AdS}}{u}\right)^4 \frac{u'^2}{f(u)}\right) d\sigma^2 \tag{5.79}$$

and it leads to the following string (Polyakov) action

$$S = \int \mathrm{d}\tau\, \mathrm{d}\rho \sqrt{\left(\frac{u}{R_{AdS}}\right)^4 \left(\rho'^2 + \frac{u'^2}{f(u)} \frac{R_{AdS}^4}{u^4}\right) (f(u) - \rho^2\omega^2)} \tag{5.80}$$

Positivity of the argument of the square root in (5.80) requires that $f(u) > \rho^2\omega^2$. This means that for a given angular frequency ω, the string solution $u(\rho)$ has to lie above the curve

$$u(\rho) \geq \frac{u_T}{(1-\rho^2\omega^2)^{1/5}} \tag{5.81}$$

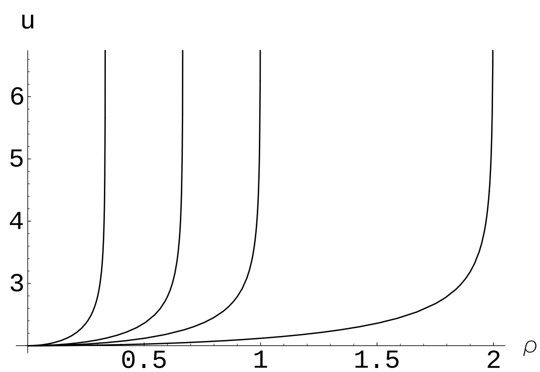

Figure 10: The boundary curve. Rotating strings have to lie above this curve in order for their action to be real. The curves correspond to the following values of the frequency ω (from right to left): 0.5, 1, 1.5 and 3. The horizon is located at $u_T = 2$.

In figure 10 these curves are depicted for various values of ω. We see that any string is allowed to touch the horizon u_T for any angular frequency ω and as ω decreases (i.e. the spin of the mesons increases) the string endpoints get more and more separated, the U-shaped string penetrates deeper to the horizon, and it becomes more and more rectangular. For a given ω, the maximal allowed extent of the string is determined by the intersection of the curve with u_0, and is given by

$$\rho_{\max} = \frac{1}{\omega}\sqrt{1 - \left(\frac{u_T}{u_0}\right)^5} \tag{5.82}$$

The equation of motion following from the action (5.80) is given by

$$
\begin{aligned}
-2\sqrt{\dots}\frac{\mathrm{d}}{\mathrm{d}\sigma}\left(\frac{1}{\sqrt{\dots}}\frac{u'}{f(u)}(f(u)-\rho^2\omega^2)\right) + f'(u)\left(\frac{u'^2\rho^2\omega^2}{f(u)^2}+\frac{u^4}{R_{AdS}^4}(\rho')^2\right) \\
+\frac{4u^3}{R_{AdS}^4}\left((\rho')^2 f(u)-(\rho')^2\rho^2\omega^2\right)=0 \quad (5.83)
\end{aligned}
$$

where $\sqrt{\dots}$ is the density of Nambu-Goto action (5.80).

The expressions for the energy and the angular momentum carried by the string are given by

$$
E=\int \mathrm{d}\sigma\,\frac{1}{\sqrt{\dots}}\left(\left(\frac{u}{R_{AdS}}\right)^4 f(u)(\rho')^2+u'^2\right) \quad (5.84)
$$

$$
J=\int \mathrm{d}\sigma\,\frac{1}{\sqrt{\dots}}\omega\rho^2\left(\left(\frac{u}{R_{AdS}}\right)^4 \rho'^2+\frac{u'^2}{f(u)}\right) \quad (5.85)
$$

The analysis of meson spectrum in [4] has showed that the meson spectrum of the SS model does not follow the well known Regge trajectories. For high-spin mesons at a fixed temperature there is a maximum value of angular momentum beyond which mesons cannot exist and have to dissociate. That is the temperature at which mesons melt is spin dependent. As the temperature increases, the maximal value of the spin that a meson can carry decreases. This behavior is also true for high-spin mesons in the AdS_6 background. Also for high mesons of fixed angular momentum, as for low-spin mesons, the energy decreases as a function of temperature.

5.4 Drag effects for quarks

In the deconfined phase the background contains a horizon and we can have a string starting on a flavor D4-brane/antibrane and going into the horizon. This string corresponds to a deconfined quark/anti-quark.

Because a strictly vertical string moving rigidly through the background would not have a real action (5.80), the string has to be bent when it is "pushed" through the plasma. In addition the bent string does not end anymore orthogonally on the brane. This means that one has to apply a force on the string endpoint, or in other words, one has to "drag" the string in order to keep it moving [24–30]. A suitable ansatz to describe

the behavior of the string that moves with speed v_x in the x direction is (in static gauge)

$$t = \tau\,, \quad u = \sigma\,, \quad x = v_x t + \xi(u) \tag{5.86}$$

Inserting (5.86) into the Nambu-Goto Lagrangian we find

$$\begin{aligned} S &= \int d^2\sigma \sqrt{-det(G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)} \\ &= \int \mathrm{d}\tau\, \mathrm{d}\rho \sqrt{1 - \frac{v_x^2}{f(u)} + \left(\frac{u}{R_{AdS}}\right)^4 f(u)\xi'^2} \end{aligned} \tag{5.87}$$

The corresponding equation for ξ implies that the conjugate momentum is a constant:

$$\pi_\xi = \frac{\partial \mathcal{L}}{\partial \xi'} = -\left(\frac{u}{R_{AdS}}\right)^4 \frac{f(u)\xi'}{\sqrt{-g}} \tag{5.88}$$

where g is the determinant of the induced metric. Inverting this relation we obtain

$$\xi' = \pi_\xi \left(\frac{R_{AdS}}{u}\right)^4 \frac{1}{f(u)} \sqrt{\frac{f(u) - v_x^2}{f(u) - \pi_\xi^2 (\frac{R_{AdS}}{u})^4}} \tag{5.89}$$

We must require that $\xi(u)$ is everywhere real, but the square root on the right hand side is in general not everywhere real. The function $f(u)$ interpolates between 1 at the boundary of AdS_6 to 0 at the horizon, so at some intermediate radius $f(u) - v_x^2$ switches sign at some intermediate point u_v, which is by definition such that $u_v^5 = u_T^5/(1 - v_x^2)$. The only way we can prevent ξ from becoming imaginary for $u < u_v$ is by choosing a value of π_ξ such that the denominator also vanishes at u_v:

$$\pi_\xi^2 = f(u_v)\left(\frac{u_v}{R_{AdS}}\right)^4 = \left(\frac{u_T}{R_{AdS}}\right)^4 \frac{v_x}{(1 - v_x^2)^{\frac{4}{5}}} \tag{5.90}$$

Plugging this back into (5.89) we find

$$\xi' = \frac{v R_{AdS}^2}{u^4} \frac{u_T^2}{f(u)} \tag{5.91}$$

Now we want to compute the σ component of the current associated with spacetime

translations along x

$$P_x^u = -G_{x\nu} g^{u\alpha} \partial X^\nu = -\frac{f(u)\xi'}{g} \left(\frac{u_T}{R_{AdS}}\right)^4 \tag{5.92}$$

where $G_{\mu\nu}$ and $g_{\alpha\beta}$ denote respectively the spacetime and induced worldsheet metric. Together with (5.87) and (5.91) it yields the drag force

$$\frac{dp}{dt} = \sqrt{-g} P_x^u = -\frac{u_T^2}{R^2} \frac{v_x^2}{(1-v_x^2)^{\frac{4}{5}}} \tag{5.93}$$

We see that there are two effects happening as one tries to move a single string in the hot background: the string shape is modified in a way which depends on the temperature and velocity, and in order to preserve the motion one needs to apply a force.

5.5 Drag effects for mesons

Now we are interested if there is a drag force on a rotating meson at finite temperature. From the condition (5.81) we can see that a simple rotating motion does not experience a drag effect because the rotating string is always sufficiently high above the curve beyond which the action would turn to be imaginary. On the other hand the bending of the rotating string does depend on the angular velocity and on the temperature.

We can also consider a linear motion of the meson in a direction orthogonal to the plane of rotation. A suitable ansatz for this motion is

$$t = \tau\,, \quad \rho = \sigma\,, \quad u = u(\rho)\,, \quad \varphi = \omega\tau\,, \quad y = v_y\,\tau \tag{5.94}$$

In this case the string action becomes

$$S = \int \mathrm{d}\tau\,\mathrm{d}\rho \sqrt{\left(\frac{u}{R_{AdS}}\right)^4 \left(1 + \frac{u'^2}{f(u)} \frac{R_{AdS}^4}{u^4}\right) \left(f(u) - \rho^2\omega^2 - v_y^2\right)}\,. \tag{5.95}$$

The only modification with respect to the rotating meson is the addition of a term "$-v_y^2$" to the last factor under the square root. The condition for the action to be real is now

$$u \geq \frac{u_T}{(1-\rho^2\omega^2 - v_y^2)^{1/5}}\,. \tag{5.96}$$

The curves are depicted in figure 11 for various values of v_y. For any high-spin meson

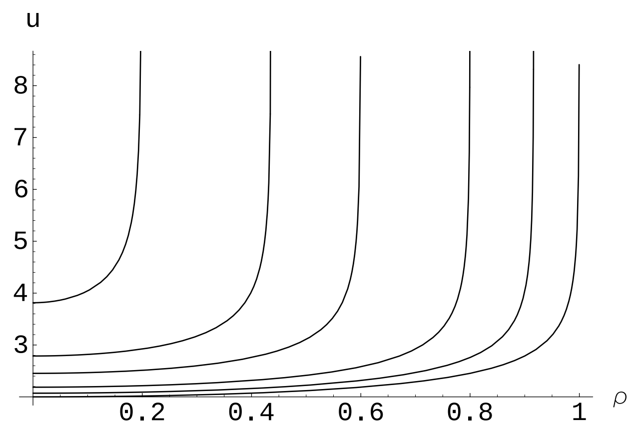

Figure 11: Analysis of the effect of a transverse velocity on the shape of spinning U-shaped strings keeping the quark masses and spin fixed. The curves display results for increasing values (from right to left) $v_y = 0, 0.4, 0.6, 0.8, 0.9, 0.98$ and $\omega = 1$. The horizon is located at $u_T = 2$.

at finite temperature one can find a generalized solution to the equation of motion such that the spinning configuration lies entirely above the curves (11). Thus, the mesons do not experience any drag effect that means that they do not experience any energy loss when propagating through the quark-gluon plasma - no force is necessary to keep them moving with a fixed velocity. In the dual language, this reflects the fact that if the quark gluon plasma is not hot enough to dissociate mesons, then these color singlets will not experience a drag force generated by a monopole interactions with the medium. However, the shape of the string in the (ρ, u) plane can be modified as it starts moving.

6 Summary

In this project we were mainly interested in the difference between HQCD models in critical and non-critical backgrounds. We looked at the thermal properties of the non-critical AdS_6 black hole background with D4-probe branes and antibranes. We found that for the chargeless flavor branes, namely, when we switched off the $\int c_5$ CS term, it has a similar behavior to the critical Sakai-Sugimoto model.

We found that the difference in free energy between Euclidean background at low and high temperatures scales as N_c^2 in the 't Hooft large N_c limit, as expected. In our model it is proportional to $(2\pi T)^5 - (1/R)^5$, while in the SS model it is proportional to $(2\pi T)^6 - (1/R)^6$. It seems that the former result fits better a five dimensional field

theory. In our model we got the chiral phase transition at the ratio of the separation distance between branes and antibranes at infinity and the radius of the x_4 circle $L/R = 1.06$, comparatively to the SS model $L/R = 0.97$. Spectrum of the low-spin mesons is discrete at low temperatures and continuous at high temperatures and we can identify the Goldstone pion associated with the spontaneous breaking of the $U(N_f)_L \times U(N_f)_R$ global chiral symmetry to the diagonal $U(N_f)$. Quarks and high-spin mesons experience a drag force at finite temperature whereas mesons do not. This is the same as was discovered in [4].

We saw that the AdS_6 and the SS model can be unified into a family of metrics assoicated with space-times of different dimensions that have a similar phase structure.

The non-critical has a serious drawback and that is the fact that the background has curvature of order one. This property implies that higher curvature corrections may be important. However, the resemblance with the critical picture indicates that presumably for the properties considered in this work the higher curvature correction are not very meaningful. On the other hand in the critical case the dilaton goes to infinity as $u \to \infty$, thus, in principle, in this region the sugra approximation is not valid and one should go to the M-theory. In the AdS_6 case dilaton is constant and we should not worry about the behavior of the model when $u \to \infty$. Again inspite of this difference the extracted physical properties seem to be alike.

Acknowledgements

We would like to thank Ofer Aharony and Stanislav Kuperstein for useful conversations. This work was supported in part by a center of excellence supported by the Israel Science Foundation (grant number 1468/06), by a grant (DIP H52) of the German Israel Project Cooperation, and by the European Network MRTN-CT-2004-512194

References

[1] O. Aharony, J. Sonnenschein and S. Yankielowicz, "A holographic model of deconfinement and chiral symmetry restoration," arXiv:hep-th/0604161.

[2] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, Prog. Theor. Phys. **113**, 843 (2005); hep-th/0412141.

[3] T. Sakai and S. Sugimoto, *More on a holographic dual of QCD*; hep-th/0507073.

[4] K. Peeters, J. Sonnenschein and M. Zamaklar, "Holographic melting and related properties of mesons in a quark gluon plasma," Phys. Rev. D **74**, 106008 (2006) [arXiv:hep-th/0606195].

[5] A. M. Polyakov, "The wall of the cave," Int. J. Mod. Phys. A **14**, 645 (1999) [arXiv:hep-th/9809057].

[6] S. Kuperstein and J. Sonnenschein, "Non-critical supergravity (d ¿ 1) and holography," JHEP **0407**, 049 (2004) [arXiv:hep-th/0403254].

[7] S. Kuperstein and J. Sonnenschein, "Non-critical, near extremal AdS(6) background as a holographic laboratory of four dimensional YM theory," JHEP **0411**, 026 (2004) [arXiv:hep-th/0411009].

[8] I. R. Klebanov and J. M. Maldacena, "Superconformal gauge theories and non-critical superstrings," Int. J. Mod. Phys. A **19**, 5003 (2004) [arXiv:hep-th/0409133].

[9] R. Casero, A. Paredes and J. Sonnenschein, "Fundamental matter, meson spectroscopy and non-critical string / gauge duality," JHEP **0601**, 127 (2006) [arXiv:hep-th/0510110].

[10] A. Armoni, E. Fuchs and J. Sonnenschein, "Confinement in 4D Yang-Mills theories from non-critical type 0 string theory," JHEP **9906**, 027 (1999) [arXiv:hep-th/9903090]. A. Giveon, D. Kutasov and O. Pelc, "Holography for non-critical superstrings," JHEP **9910**, 035 (1999) [arXiv:hep-th/9907178]. G. Ferretti, J. Kalkkinen and D. Martelli, "Non-critical type 0 string theories and their field theory duals," Nucl. Phys. B **555**, 135 (1999) [arXiv:hep-th/9904013]. G. Ferretti and D. Martelli, "On the construction of gauge theories from non critical type 0 strings," Adv. Theor. Math. Phys. **3**, 119 (1999) [arXiv:hep-th/9811208]. E. Alvarez and C. Gomez, "Non-critical confining strings and the renormalization group," Nucl.

Phys. B **550**, 169 (1999) [arXiv:hep-th/9902012]. K. Ghoroku, "Yang-Mills theory from non-critical string," J. Phys. G **26**, 233 (2000) [arXiv:hep-th/9907143]. S. K. Ashok, "Yang-Mills theories from non-critical strings and the statistics of type G. Bertoldi and N. Dorey, "Non-critical superstrings from four-dimensional gauge theory," JHEP **0511**, 001 (2005) [arXiv:hep-th/0507075]. F. Bigazzi, R. Casero, A. L. Cotrone, E. Kiritsis and A. Paredes, "Non-critical holography and four-dimensional CFT's with fundamentals," JHEP **0510**, 012 (2005) [arXiv:hep-th/0505140]. S. K. Ashok, S. Murthy and J. Troost, "D-branes in non-critical superstrings and minimal super Yang-Mills in Nucl. Phys. B **749**, 172 (2006) [arXiv:hep-th/0504079]. A. Fotopoulos, V. Niarchos and N. Prezas, "D-branes and SQCD in non-critical superstring theory," JHEP **0510**, 081 (2005) [arXiv:hep-th/0504010]. D. Israel, "Non-critical string duals of N = 1 quiver theories," JHEP **0604**, 029 (2006) [arXiv:hep-th/0512166]. A. R. Lugo and M. B. Sturla, "Space-time filling branes in non critical (super) string theories," arXiv:0709.0471 [hep-th]. S. Murthy and J. Troost, "D-branes in non-critical superstrings and duality in N = 1 gauge theories with flavor," JHEP **0610**, 019 (2006) [arXiv:hep-th/0606203]. V. Niarchos, "Non-critical superstrings and supersymmetric QCD," Fortsch. Phys. **54**, 443 (2006). F. Bigazzi, R. Casero, A. Paredes and A. L. Cotrone, "Non-critical string duals of four-dimensional CFTs with fundamental matter," Fortsch. Phys. **54**, 300 (2006). S. K. Ashok, S. Murthy and J. Troost, "D-branes in unoriented non-critical strings and duality in SO(N) and Sp(N) gauge theories," JHEP **0706**, 047 (2007) [arXiv:hep-th/0703148]. U. Gursoy and E. Kiritsis, "Exploring improved holographic theories for QCD: Part I," arXiv:0707.1324 [hep-th].

[11] U. Gursoy, E. Kiritsis and F. Nitti, "Exploring improved holographic theories for QCD: Part II," arXiv:0707.1349 [hep-th].

[12] A. Schwimmer and S. Theisen, "Universal features of holographic anomalies," JHEP **0310**, 001 (2003) [arXiv:hep-th/0309064].

[13] J. Sonnenschein " Non critiacl gauge/gravity duality" published in "Einstein in Alexandria - The Scientific symposium", Editor E. Witten Alexandria, 2006

[14] A. Parnachev and D. A. Sahakyan, "Chiral phase transition from string theory," arXiv:hep-th/0604173.

[15] N. Horigome and Y. Tanii, arXiv:hep-th/0608198.

[16] E. Witten, "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].

[17] J. M. Maldacena, "Wilson loops in large N field theories," Phys. Rev. Lett. **80**, 4859 (1998) [arXiv:hep-th/9803002].

[18] S. J. Rey and J. T. Yee, "Macroscopic strings as heavy quarks in large N gauge theory and anti-de Eur. Phys. J. C **22**, 379 (2001) [arXiv:hep-th/9803001].

[19] Y. Kinar, E. Schreiber and J. Sonnenschein, "Q anti-Q potential from strings in curved spacetime: Classical results," Nucl. Phys. B **566**, 103 (2000) [arXiv:hep-th/9811192].

[20] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-th/9802042].

[21] F. Neri and A. Gocksch, "Chiral Symmetry Restoration In Large N QCD At Finite Temperature," Phys. Rev. D **28**, 3147 (1983).

[22] R. D. Pisarski, "Finite Temperature QCD At Large N," Phys. Rev. D **29**, 1222 (1984).

[23] M. Kruczenski, L. A. P. Zayas, J. Sonnenschein and D. Vaman, "Regge trajectories for mesons in the holographic dual of large-N(c) QCD," JHEP **0506**, 046 (2005) [arXiv:hep-th/0410035].

[24] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, "Energy loss of a heavy quark moving through N = 4 supersymmetric Yang-Mills plasma," JHEP **0607**, 013 (2006) [arXiv:hep-th/0605158].

[25] J. Casalderrey-Solana and D. Teaney, "Heavy quark diffusion in strongly coupled N = 4 Yang Mills," arXiv:hep-ph/0605199.

[26] S. S. Gubser, "Drag force in AdS/CFT," Phys. Rev. D **74**, 126005 (2006) [arXiv:hep-th/0605182].

[27] C. P. Herzog, "Energy loss of heavy quarks from asymptotically AdS geometries," arXiv:hep-th/0605191.

[28] E. Caceres and A. Guijosa, "Drag force in charged N = 4 SYM plasma," arXiv:hep-th/0605235.

[29] J. J. Friess, S. S. Gubser and G. Michalogiorgakis, "Dissipation from a heavy quark moving through N = 4 super-Yang-Mills plasma," arXiv:hep-th/0605292.

[30] S. J. Sin and I. Zahed, "Ampere's law and energy loss in AdS/CFT duality," arXiv:hep-ph/0606049.